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Polarized parton distribution in the relativistic quark exchange framework

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Abstract. The Spin dependent gluon and sea quark distributions of the proton and the neutron are extracted in the leading order (LO) and the next-to-leading order (NLO) QCD. The relativistic quark exchange model is used to calculate the related valence quark spin dependent structure function. The inverse Mellin transform technique is performed to evaluate the polarized x-dependent distributions of the gluon and the sea quark from the various moments of the valence quarks. It is shown that the calculated spin structure functions (SSF) of the proton and the neutron are in good agreement with the available data, such as E143, SMC, E142, E154 and Hermes experiments. A comparison is also made with the other theoretical models. Finally it is shown that the above calculated parton distributions improve the SSF of the proton and the neutron.

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1 Introduction

Deep inelastic scattering of polarized leptons from polarized nucleons provides the most important information on the nucleons spin dependent structure functions (SSF). In the past few years there have been many measurements on the nucleon SSF [1–3]. Recently this has been also extracted by deep inelastic scattering of polarized leptons off polarized nuclear targets, e.g. deuteron (SMC [1]) and ^3He (E142, E154 and Hermes experiments [3]). Therefore information about SSF of the neutron can in principle depend upon nuclear structure effects as well.

On the other hand, not only the uncertainties in the experimental data and their corresponding statistical errors have been substantially reduced but also the values of data are available at small x as well as different scales [4]. This suggests that one can analyze polarized deep inelastic lepton scattering within the framework of radiative parton model which in turns bring in the calculation of various moments of SSF as well as its first moment.

On the theoretical side, how the spin of the hadron is shared among its parton constituents and the evaluation of the nucleon structure functions is still an outstanding problem in high energy hadronic physics [5]. A most natural possibility is to build the nucleons entirely from the valence quarks [6] at some resolution scale, μ_0 (static point). But it is known that at high energies this picture is no longer valid [7]. Since the gluons are then generated through bremsstrahlung off the constituent quarks. Part of the so produced gluons materialize into the quark-antiquarks pairs (the sea quarks [8]). Because of these degrees of freedom, as we pointed out before, recently attention has been paid to the other moments of the polarized parton distribution [9]. So it is appropriate to perform a detail study of the polarized parton distribution by using the leading-order and the next-to-leading order evaluation procedure of the renormalization-group equations to generate the gluon and the sea contents at a new scale $Q^2 > \mu_0^2$ [9].

For the valence quark, most of the authors have used (*i*) the fitting procedure by imposing various sum rules construction [10] or (*ii*) the unrealistic approach such as the instantons [11], Isgur-Karl [11], etc models or (*iii*) for the unpolarized case, by using two-field operators correlation functions to define the quark and gluon distributions [12]. But in this article, we intend to use a realistic formalism which was originally introduced by Hoodbhoy and Jaffe (HJ) [13,14] to investigate the multi-quark exchange in the nuclear system. This model was successfully applied by us to light nuclei [15] and nuclear matter [16] to calculate the quark distribution and the nucleus structure function (the EMC-effect) respectively. In our recent work we developed HJ method to calculate the SSF of three nucleon systems (^3H and ^3He) as well as the proton and the neutron by using the above formalism and the familiar convolution approach [17]. In this framework we intend to investigate the effect of higher order corrections to the valence quark structure of different nuclear target.

So the paper is organized as following. We begin Sect. 2 by introducing the polarized parton distributions and the inverse Mellin transformation. In Sect. 3 we apply the quark-exchange formalism to calculate the spin dependent valence quark momentum distribution in the three nucleon systems. Finally numerical results and conclusion are presented in Sect. 4.

2 Polarized parton distributions

We start by the radiative generation technique proposed [18] for explaining the magnitude and the sign of the total gluon and the sea quark polarization. It is well known that the Q^2 dependence implied by QCD could be simply expressed in terms of the parton density moments. One therefore can write in the N-moment space [19],

$$\Delta M_{\mathcal{P}}(n,Q^2) = \int_0^1 x^{n-1}\Delta\mathcal{P}(x,Q^2)\,dx \qquad (1)$$

where $\mathcal{P} = q^v, \bar{q}, G$ and $\Delta M_{\mathcal{P}}(n,Q^2)$ is the Mellin transform of the polarized parton distribution, $\Delta\mathcal{P}(x,Q^2)$. We take an SU(3)-flavor symmetric polarized sea in which $\Delta\bar{q} = \Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s} = \Delta s$.

As it was stated before, it is assumed that at a static point the gluon and the anti-quark distributions should satisfy the following boundary conditions at some low scale μ_0^2, respectively,

$$\Delta G(x,\mu_0^2) = 0 \qquad \Delta\bar{q}(x,\mu_0^2) = 0 \qquad (2)$$

Then the leading-order (LO) generation of the total sea and the gluon can be given by the bremsstrahlung radiation from the valence quarks of different flavors. They are obtained as a solution of the polarized evolution equation in the N-moment space [20,21],

$$\begin{aligned}\sum_{q=u,d}\Delta M_q^v(n,Q^2) &= \{\sum_{q=u,d}\Delta M_q^v(n,\mu_0^2)\}L^{-a^n_{NS}}\\ \Delta M_{\bar{q}}(n,Q^2) &= \frac{1}{6}\{\sum_{q=u,d}\Delta M_q^v(n,\mu_0^2)\}\\ &\times(\alpha^n L^{-a^n_-} + (1-\alpha_n)L^{-a^n_+} - L^{-a^n_{NS}})\\ \Delta M_G(n,Q^2) &= \{\sum_{q=u,d}\Delta M_q^v(n,\mu_0^2)\}\\ &\times\frac{\alpha^n(1-\alpha^n)}{\beta^n}(L^{-a^n_-} - L^{-a^n_+})\end{aligned} \qquad (3)$$

where $a_i^n = -2\frac{\Delta P_i^n}{\beta_0}$ and β_0, α^n and β^n are written as following,

$$\begin{aligned}\beta_0 &= (11 - \frac{2}{3}N_f)\\ \alpha^n &= \frac{\Delta P^n_{qq} - \Delta P^n_+}{\Delta P^n_- - \Delta P^n_+}\\ \beta^n &= \frac{\Delta P^n_{qg}}{\Delta P^n_- - \Delta P^n_+}\end{aligned} \qquad (4)$$

N_f is the number of active quark flavors (in our case $N_f = 3$) and

$$\begin{aligned}\Delta P^n_\pm &= \frac{1}{2}[\Delta P^n_{qq} + \Delta P^n_{gg}\\ &\pm\sqrt{(\Delta P^n_{gg} - \Delta P^n_{qq})^2 + 4\Delta P^n_{qg}\Delta P^n_{gq}}]\\ \Delta P^n_{NS} &= \Delta P^n_{qq} = \frac{4}{3}[\frac{3}{2} + \frac{1}{n(n+1)} - 2S_1(n)]\\ \Delta P^n_{qg} &= N_f\frac{(n-1)}{n(n+1)}\\ \Delta P^n_{gq} &= \frac{4}{3}\frac{(n+2)}{n(n+1)}\\ \Delta P^n_{gg} &= 3[\frac{11}{6} - \frac{N_f}{9} + \frac{4}{n(n+1)} - 2S_1(n)]\end{aligned} \qquad (5)$$

with

$$\begin{aligned}S_1(n) &= \psi(n+1) + \gamma_E,\\ \psi(n) &= \frac{d}{dn}\ln\Gamma(n), \qquad (6)\\ \gamma_E &= 0.5772 \qquad (7)\end{aligned}$$

where γ_E is the Euler's constant [9].

By using the different polarized valence quark distribution $\Delta q^v(x,Q^2)$, to be discussed later on according to the quark exchange formalism, $\Delta M_q^v(n,Q^2)$ will be calculated through (1) where the quantity L, the coupling ratio, is defined as (Λ^2 is QCD cut off parameter),

$$L = \frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)} = \frac{\ln(\frac{Q^2}{\Lambda^2})}{\ln(\frac{\mu_0^2}{\Lambda^2})} \qquad (8)$$

On the other hand, the second moment of the nucleon (i.e. the proton and the neutron in average) structure function is related to L [22] according to following equation:

$$\begin{aligned}M^N(n,Q^2) &= \int_0^1 F_2^N(x,Q^2)\,dx\\ &= \frac{2}{9}[\frac{9}{25} + \frac{16}{25}L^{-\frac{50}{81}}] + \frac{1}{18}L^{-\frac{32}{81}}\end{aligned} \qquad (9)$$

where $F_2^N(x,Q^2)$ is the unpolarized nucleon structure function. It should be emphasized that the results of the evaluation should not depend on Λ and μ_0, but only on their combination through (7) i.e. L [8]. Experimentally [23] $M^N(n,Q^2) = 0.127$ at $Q^2 = 15\ GeV^2$. So one can calculate L at $Q^2 = 4\ GeV^2$ according to (7) and (8) which leads to $L = 10.3$ ($\mu_0 = 0.266$ GeV with $\Lambda = 0.232$ GeV). All of the above solutions in the N-moment space (1)–(8) can be inverted into the x-space by the inverse Mellin ($\mathcal{M}^{-1}$) transformation of (1) [24] i.e.

$$\begin{aligned}\Delta\mathcal{P}(x,Q^2) &= \mathcal{M}^{-1}[\Delta M_{\mathcal{P}}(n,Q^2)]\\ &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-n}\Delta M_{\mathcal{P}}(n,Q^2)\,dn\end{aligned} \qquad (10)$$

The value of real number c is chosen such that the contour passes through the right hand side of any singularities of $\Delta M_{\mathcal{P}}(n,Q^2)$.

In order to calculate the inverse Mellin transforms numerically (9), we should rewrite this equation as,

$$\Delta\mathcal{P}(x,Q^2)=\frac{1}{\pi}\int_0^\infty \mathcal{I}m[\exp(i\phi)x^{-c-z\exp(i\phi)} \times \Delta M_{\mathcal{P}}(c+z\exp(i\phi),Q^2)]\ dz \quad (11)$$

In general the result of above integral should not depend on c and ϕ, but a good selection of the c and ϕ parameters cause an efficient numerical calculation [24]. Then, by using the above numerical inverse Mellin transformation we could calculate $\Delta G(x,Q^2)$ and $\Delta\bar{q}(x,Q^2)$ for the proton and the neutron, according to (3).

3 Quark exchange formalism

Now, let us discuss the evaluation of valence quark distribution as we pointed out before. We assume the nucleons are composed of three valence quarks in the following way:

$$|\alpha\rangle=\mathcal{N}^{\alpha^\dagger}|0\rangle=\frac{1}{\sqrt{3!}}\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}q^\dagger_{\mu_1}q^\dagger_{\mu_2}q^\dagger_{\mu_3}|0\rangle \quad (12)$$

where α designate the nucleon states $\{\boldsymbol{P},M_S,M_T\}$ and μ stands for the quark states $\{\boldsymbol{k},m_s,m_t,c\}$. With the convention that there is a summation on the repeated indices as well as integration over $\boldsymbol{k}$. $q^\dagger$ ($\mathcal{N}^{\alpha^\dagger}$) are the creation operators for quarks (nucleons) and $\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}$ are the totally antisymmetric nucleon wave function, i.e.

$$\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}=D(\mu_1,\mu_2,\mu_3;\alpha_i) \times\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3-\boldsymbol{P})\phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{P}) \quad (13)$$

$D(\mu_1,\mu_2,\mu_3;\alpha_i)$ depend on the Clebsch-Gordon coefficients $C^{j_1j_2j}_{m_1m_2m}$ and the color factor $\epsilon_{c_1c_2c_3}$,

$$D(\mu_1,\mu_2,\mu_3;\alpha_i)=\frac{1}{\sqrt{3!}}\epsilon_{c_1c_2c_3} \cdot\frac{1}{\sqrt{2}}\sum_{s,t=0,1}C^{\frac12 s\frac12}_{m_{s_\sigma}m_sM_{S_{\alpha_i}}}C^{\frac12\frac12 s}_{m_{s_\mu}m_{s_\nu}m_s} \times C^{\frac12 t\frac12}_{m_{t_\sigma}m_tM_{T_{\alpha_i}}}C^{\frac12\frac12 t}_{m_{t_\mu}m_{t_\nu}m_t} \quad (14)$$

$\phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{P})$ is the nucleon wave function and we write it in a Gaussian form (b $\simeq$ nucleons radius) :

$$\phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{P})=\left(\frac{3b^4}{\pi^2}\right)^{\frac34}\exp[-b^2\frac{(k_1^2+k_2^2+k_3^2)}{2}+\frac{b^2P^2}{6}] \quad (15)$$

We can define the nucleus state based on nucleon creation operators, i.e.

$$|\mathcal{A}_i=3\rangle=(3!)^{-\frac12}\chi^{\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1\dagger}\mathcal{N}^{\alpha_2\dagger}\mathcal{N}^{\alpha_3\dagger}|0\rangle \quad (16)$$

where $\chi^{\alpha_1\alpha_2\alpha_3}$ is a complete antisymmetric nuclear wave function (it is taken from Faddeev calculation with Reid soft core potential [25]) and it should be interpreted as the wave function governing the center of mass motion of the three quark clusters.

The quark momentum distributions for quarks with fixed flavor and spin polarization in a three nucleon system are defined as,

$$\rho_{\bar\mu}(\boldsymbol{k};\mathcal{A}_i)=\frac{\langle\mathcal{A}_i=3|q^\dagger_{\bar\mu}q_{\bar\mu}|\mathcal{A}_i=3\rangle}{\langle\mathcal{A}_i=3|\mathcal{A}_i=3\rangle} \quad (17)$$

where the sign bar means no summation on m_s,m_t and integration over $\boldsymbol{k}$ in the μ indices. By using the above definition we can calculate the quark polarized momentum distribution for each flavor as below,

$$\Delta\rho_q(\boldsymbol{k};\mathcal{A}_i)=\sum_{q=u,d,j=1-4}M_{qj}\exp(-a_j\boldsymbol{k}^2) \quad (18)$$

where

$$\Delta\rho_q(\boldsymbol{k};\mathcal{A}_i)=\rho_{q\uparrow}(\boldsymbol{k};\mathcal{A}_i)-\rho_{q\downarrow}(\boldsymbol{k};A_i) \quad (19)$$

The matrix representation of polarized quark momentum distribution for three nucleon systems is ($\mathcal{F}=\frac{b^3}{1+0.552I}$):

$$\begin{pmatrix}\Delta\rho_u(\boldsymbol{k};{}^3H)\\ \Delta\rho_d(\boldsymbol{k};{}^3H)\\ \Delta\rho_u(\boldsymbol{k};{}^3He)\\ \Delta\rho_d(\boldsymbol{k};{}^3He)\end{pmatrix} =\mathcal{F}\begin{pmatrix}0.367 & -0.313I & 1.612I & -0.026I\\ -0.201 & 0.162I & 0.601I & 0.026I\\ -0.201 & 0.162I & 0.601I & 0.026I\\ 0.367 & -0.313I & 1.612I & -0.026I\end{pmatrix} \times\begin{pmatrix}\exp(-\frac32 b^2\boldsymbol{k}^2)\\ \exp(-\frac32 b^2\boldsymbol{k}^2)\\ \exp(-\frac{12}{7}b^2\boldsymbol{k}^2)\\ \exp(-3b^2\boldsymbol{k}^2)\end{pmatrix} \quad (20)$$

where [13,15]

$$I=8\pi^2\int_0^\infty x^2\ dx\int_0^\infty y^2\ dy \times\int_{-1}^1 d(cos\theta)\exp[-\frac{3x^2}{4b^2}]|\chi(x,y,cos\theta)|^2$$

In above equation $\chi(x,y,cos\theta)$ is the Fourier transform of the nucleus wave function and the parameter b is fixed such that we could get the correct normalization and the charge radius of ^3He and ^3H i. e. 1.68 fm and 1.56 fm corresponding to $b\simeq .837$ fm and $b\simeq .780$ fm respectively (we have only considered the leading order expansion in $\chi(\boldsymbol{x},\boldsymbol{y})$ [7,13,17]).

4 Results and discussions

The polarized valence parton distribution at Q^2, can be related to the polarized momentum distribution for each flavor in the nucleus, by considering the relativistic corrections according to the following equation [17,26],

$$\Delta q^v(x,Q^2;\mathcal{A}_i)=\frac{1}{(1-x)^2}\int \Delta\rho_q(\boldsymbol{k};\mathcal{A}_i)\delta(\frac{x}{(1-x)}-\frac{k_+}{M})\,d\boldsymbol{k} \quad (21)$$

By performing the angular integration, we find,

$$\Delta q^v(x,Q^2;\mathcal{A}_i) = \frac{2\pi M}{(1-x)^2}\int_{k_{min}}^{\infty} \Delta\rho_q(\boldsymbol{k};\mathcal{A}_i)k\ dk \quad (22)$$

with

$$k_{min}(x) = \frac{(\frac{xM}{1-x}+\epsilon_0)^2 - m^2}{2(\frac{xM}{1-x}+\epsilon_0)} \quad (23)$$

m (M) is the quark (nucleon) mass , k_+ is the light-cone momentum of initial quark and ϵ_0 is the quark binding energy. Their numerical values at $Q^2 = 4\ GeV^2$ are m = 180 MeV and ϵ_0=215 MeV [17].

A simple approach for describing the scattering processes involving the nucleus is convolution model. It assumes that the scattering may be described in terms of incoherent scattering off nuclear constituents [27]. So we can define the valence quark distribution of the bound nucleon in terms of valence quark distribution of the free nucleon as below,

$$\Delta q^v(x,Q^2;\mathcal{A}_i)=a\sum_N\int \Delta q^v(\frac{x}{y_{\mathcal{A}_i}},Q^2;N)f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})\,dy_{\mathcal{A}_i} \quad (24)$$

$f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})$ is the nucleon momentum distribution in the nucleus, where it is approximated it by a Fermi gas distribution and a is the nuclear asymmetry [27] (in case of $j=\frac{1}{2}$, a=1). If the nucleon momentum distribution is sharp we can expand (23) around $\frac{x}{\langle y_{\mathcal{A}_i}\rangle}$ with $\langle y_{\mathcal{A}_i}\rangle = 1+\frac{\bar\epsilon}{M}$ and $\bar\epsilon$ as the average removal energy of the nucleon, then

$$\Delta q^v(\frac{x}{\langle y_{\mathcal{A}_i}\rangle},Q^2;N) = \Delta q^v(x,Q^2;\mathcal{A}_i) \quad (25)$$

Although with a very good approximation one can consider ^3H and ^3He as the proton and the neutron targets [3] respectively.

The general form for the parameterization of the polarized parton distributions (including the valence quark) [20,10,28] is:

$$x\ \Delta\mathcal{P}(x,Q^2) = A_{\mathcal{P}}\eta_{\mathcal{P}}x^{a_{\mathcal{P}}}(1-x)^{b_{\mathcal{P}}}(1+\gamma_{\mathcal{P}}x+\varrho_{\mathcal{P}}x^{\frac{1}{2}}) \quad (26)$$

where the normalization factor $A_{\mathcal{P}}$ is defined as (Beta(x,y) are the familiar Beta functions which are defined in terms of Gamma functions),

$$A_{\mathcal{P}}^{-1} = \left(1+\gamma_{\mathcal{P}}\frac{a_{\mathcal{P}}}{a_{\mathcal{P}}+b_{\mathcal{P}}+1}Beta(a_{\mathcal{P}}+b_{\mathcal{P}}-1,b_{\mathcal{P}}+1)\right) + \varrho_{\mathcal{P}}Beta\left(a_{\mathcal{P}}+b_{\mathcal{P}}-\frac{1}{2},b_{\mathcal{P}}+1\right) \quad (27)$$

In the above equation the first moment is given by $\eta_{\mathcal{P}}$. The result of above parameterization for the valence quarks

Table 1. The parameters of parton distributions for the proton according to the equation (22)

	Δu^v	Δd^v	ΔG	$\Delta\bar q.$
$\eta_{\mathcal{P}}$	.7978	-.4822	1.314	-.0028
$a_{\mathcal{P}}$	1.146	1.027	0.315	.44
$b_{\mathcal{P}}$	3.423	2.958	8.	12.32
$\gamma_{\mathcal{P}}$	-4.4943	-2.459	0	-1.7
$\varrho_{\mathcal{P}}$	2.598	1.995	-3.56	-7.

Table 2. The parameters of parton distributions for the neutron according to the equation (22)

	Δu^v	Δd^v	ΔG	$\Delta\bar q.$
$\eta_{\mathcal{P}}$	-.4786	.7958	1.305	-.00324
$a_{\mathcal{P}}$	1.0881	1.1391	0.3127	0.454
$b_{\mathcal{P}}$	3.643	3.891	8.	12.91
$\gamma_{\mathcal{P}}$	-1.86	-5.11	0	-1.71
$\varrho_{\mathcal{P}}$	1.338	2.944	-3.743	-7.11

(21), the gluon and the sea quarks (equation 10) in the proton and the neutron are given in Tables 1 and 2, respectively. Unlike of the others calculations our parameters for the quarks are fixed from the beginning and have been calculated in a consistent ways.

In Figs. 1 and 2 we present $x\Delta G(x,Q^2 = 4GeV^2)$ and $x\Delta\bar q(x,Q^2 = 4GeV^2)$, for the proton and the neutron respectively. The results of others calculations are given for comparison. The GS(LO, Mellin) is the result of our calculation for the proton but by using the valence quark distribution of reference [10]. As we pointed out before, GS [10] and GRSV [30] have used various sum rules

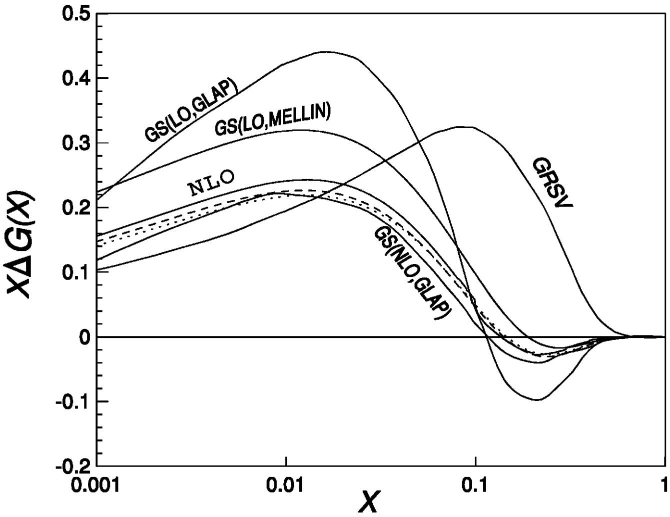

Fig. 1. $x\Delta G(x)$ at $Q^2 = 4GeV^2$ for the proton (dotted curve,LO), (full curve, NLO) and the neutron (dashed curve,LO). The full curves are the result of references [10] (GS(LO, GLAP) and GS(NLO,GLAP)) and [29] (GRSV). GS(LO, Mellin) is the result of our calculation with the valence quark distribution of reference [10]

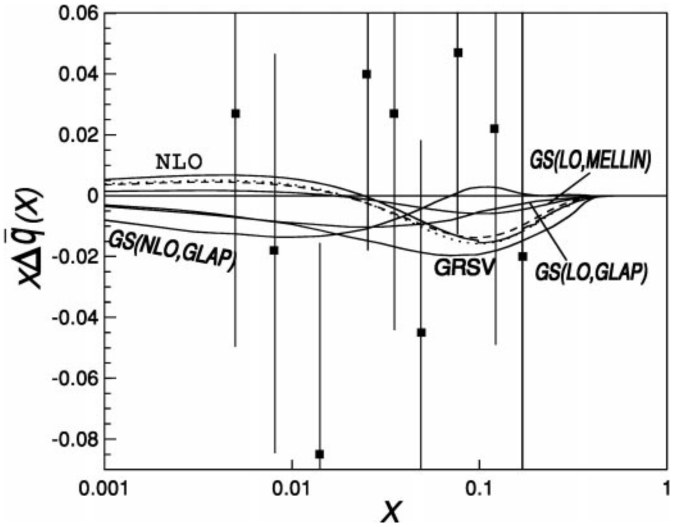

Fig. 2. As figure 1 but for $x\Delta\bar{q}(x)$. The full box shows the experimental data of SMC group [1]

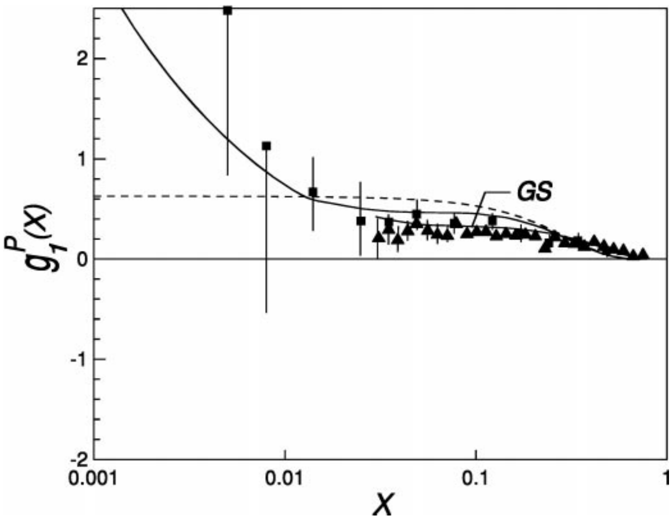

Fig. 3. Comparison of $g_1^p(x)$ with experimental data: SMC [1] (full box) and E143 [2] (full triangle). Dashed curve (the valence quark only), full curve (with sea quarks)

or some kind ansatz for the valence quark distribution. However our result is in good agreement with those of GS rather than GRSV who get the gluons pick at larger x values. The quantity $\Delta M_G^p(1, 4GeV^2) = 1.314$, is comparable with those of Glück et al [9] $\Delta M_G(1, 4GeV^2) = 1.509$ and 1.361 [30] and Ball et al [31] 1.50 ± 0.8 and 1.30 ± 0.56 [32].

Up to now we have not performed the next-to-leading order corrections to the above calculated parton distribution functions. We know that the NLO contributions are negligible when the QCD coupling constant is small. But on the other hand for the large QCD coupling constant, where the NLO corrections are sizable, the definition of polaraized structure functions in terms of parton distribution functions become scheme-dependence beyond LO. However in order to get some estimate of NLO corrections, we give the results of the NLO effect on our above LO calculations. In this respect, we follow the formalism which have been given by Ball et al. [31,32]. In this framework the LO predictions are modified as,

$$\Delta f^{NLO}(x, Q^2) = [1 + \epsilon_f(\frac{\rho}{\gamma_f})^3(\alpha_s(Q_0^2) - \alpha_s(Q^2))] \times \Delta f^{LO}(x, Q^2)$$

where

$$\Delta f(x, Q^2) = v^{\pm}(x, Q^2) \quad or \quad \Delta q_{NS}(x, Q^2)$$

$$\epsilon_{NS} = \frac{8}{3\pi\beta_0}$$

$$\epsilon_{\pm} = \frac{112}{3\pi\beta_0}[(1 - \frac{n_f}{14}) \pm \frac{13}{14}(1 - \frac{11n_f}{104})/\sqrt{(1 - \frac{3n_f}{32})}]$$

$$\rho = \frac{\xi}{\zeta}, \xi = ln(\frac{x_0}{x}), \zeta = ln\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}$$

$$\gamma_{NS}^2 = \frac{8}{33 - 2n_f}, \gamma_{\pm}^2 = \gamma_{NS}^2(5 \pm 4\sqrt{(1 - \frac{3n_f}{32})}$$

and x_0 is a reference value of x such that the approximation of the anomalous dimensions is valid for $x \leq x_0$ and $Q^2 \geq Q_0^2$ (in our case $x_0 = 0.1$). In above equations,

$$v^{\pm} = \Delta\Sigma + C^{\pm}\Delta G, C^{\pm} = 2(1 \pm \sqrt{(1 - \frac{3n_f}{32}}))$$

and $\Delta\Sigma$ and Δq_{NS} have their usual definition [30–32]. The NLO corrections to our LO parton distributions are also displayed in Figs. 1 and 2. It is seen that there is not much difference between our NLO and LO results which is in agreement with those of GRSV [30]. But the GS calculations shows larger effect due to the next-to-leading order contributions.

According to the leading order QCD parton model the polarized structure function of the nucleons at given Q^2 with the leading order contributions can be expressed in terms of the various polarized parton distributions i.e. [29]

$$g_1(x; Q^2) = \frac{1}{2} \sum_{q=u,d,s} e_q^2\{\Delta q^v(x, Q^2) + \Delta\bar{q}(x, Q^2)\} \quad (28)$$

The results of our calculation according to the above equation i.e. $g_1^p(x)$ and $g_1^n(x)$ are given in Figs. 3 and 4, respectively. Dashed curve represents only the contribution of valence quarks to the SSF [17]. It is seen that the sea quarks improve our previous calculation and our results get closer to the experimental data. The calculations of GS [10] and GRSV [30] are also given for comparison.

In Table 3, the first moments i.e. Γ_1^p and Γ_1^n are compared with corresponding experimental data. Again, it is seen that the sea-quarks improve our previous result and we are very close to the experimental predications.

Figures 5 and 6 show the comparison of xg_1^p and xg_1^n against experimental data as well. The improvement respect to the experimental data is evident for both the proton and the neutron structure functions and it is indi-

Table 3. The Comparison $\Gamma_1^i = \int_0^1 g_1^i(x)\ dx$ with the experimental data

	$Q^2(GeV^2)$	Γ_1^p	Γ_1^n
Present Calculation	4	0.1347	-0.04839
Valence Quark only	4	0.1420	-0.05091
SMC	10	$0.136 \pm 0.011 \pm 0.11$	
E143	3	$0.127 \pm 0.004 \pm 0.010$	
E142	2		-0.022 ± 0.011
E154	5		$-.037 \pm 0.004 \pm 0.010$
HERMES	2.5		$-.037 \pm 0.013 \pm 0.011$

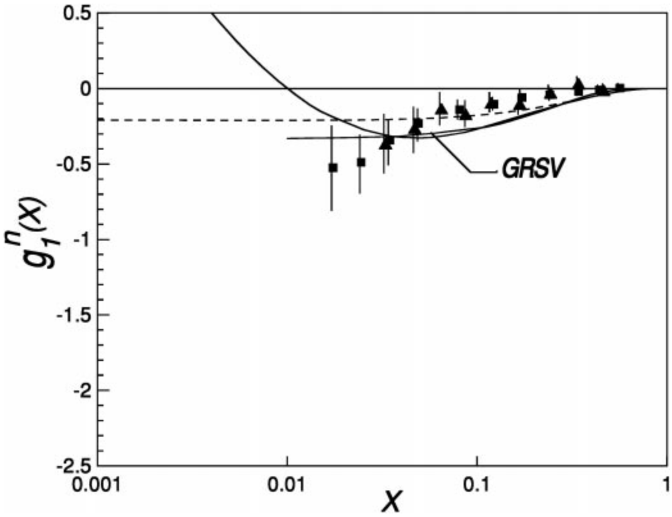

Fig. 4. As Fig. 3 but for neutron i.e. comparison of $g_1^n(x)$ with experimental data: Hermes [3] (full box) and E154 [3] (full triangle)

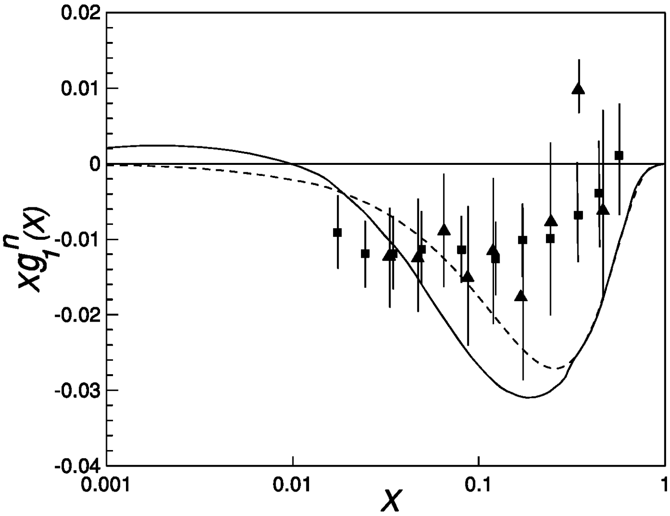

Fig. 6. As Fig. 4 but for $xg_1^n(x)$

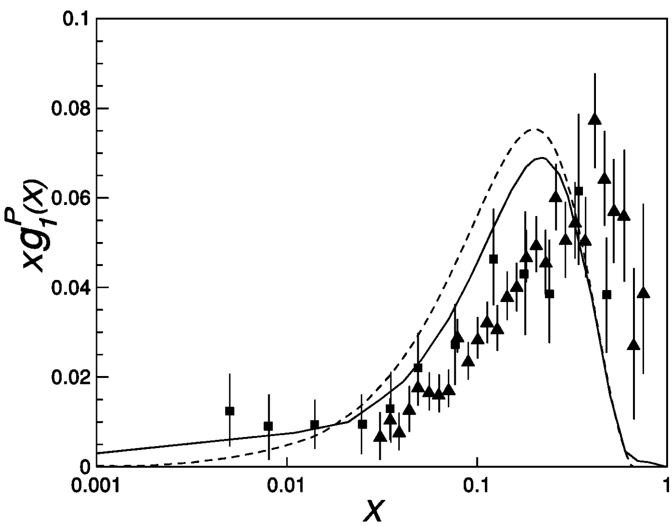

Fig. 5. As Fig. 3 but for $xg_1^p(x)$

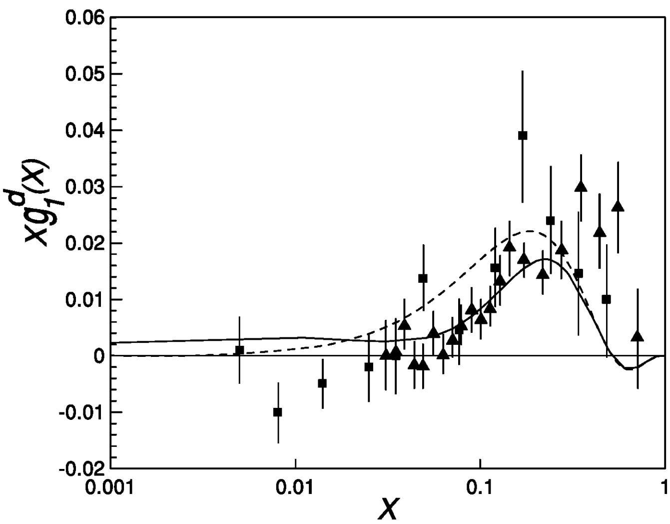

Fig. 7. As Fig. 5 but for deuteron

cating that the inclusion of the sea-quarks are very important and the LO corrections are needed to get quatitative results.

In Fig. 7 we present the result of our calculation for deuteron, $xg_1^d(x)$ at $Q^2 = 4\ GeV^2$. This has been calculated from the following equation,

$$xg_1^d(x, Q^2) = \frac{1}{2}x[g_1^p(x, Q^2) + g_1^n(x, Q^2)]R^{\frac{d}{N}}(x)$$

where $R^{\frac{d}{N}}(x)$=0.892 and it is valid for $0.001 \leq x \leq 0.7$

according to the reference [21]. Full curve is the calculation with the sea quarks and dashed curve shows only the valence quark contributions. The corresponding experimental data is also displayed [1,2].

To end this paper let us summarize and conclude as following. We have performed a detailed study of spin-dependent structure function in the framework of relativistic quark exchange model and a consistent leading-order QCD calculations. We have adopted this point view that there is no significant polarization of the gluons and the sea quarks at low Q^2. We have ignored the next-leading-order correction in our structure functions which should be negligible at least at $x \geq 5 \times 10^{-3}$ [30].

Our numerical analysis has revealed that by including the sea-quarks distributions in a relativistic quark model, as the one we have used here, it is possible to predict the available experimental data.

Future measurements at higher energies are needed to determine the behavior of the structure function at low x and the exact rules of the gluons and the sea-quarks in the hadrons.

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